LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

UCRL-PROC-207348

Application of the Extended Pairing Model to Heavy Isotopes

V. G. Gueorguiev, F. Pan, J. P. Draayer

October 20, 2004

The Fourth International Conference on Exotic Nuclei and Atomic Masses
Pine Mountain, GA, United States
September 12, 2004 through September 16, 2004

EPJ manuscript No.
(will be inserted by the editor)

Application of the Extended Pairing Model to Heavy Isotopes

V. G. Gueorguiev[1,2], Feng Pan[3], and J. P. Draayer[2]

[1] Lawrence Livermore National Laboratory, Livermore, CA 94550, USA
[2] Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA
[3] Department of Physics, Liaoning Normal University, Dalian, 116029, P. R. China

UCRL-PROC-207348

Abstract. The binding energies of three isotopic chains ($^{100-130}$Sn, $^{152-181}$Yb, and $^{181-202}$Pb) have been studied using the exactly solvable extended pairing model [1]. The unique pairing strength $G(A)$, which reproduces the experimental binding energies, has been determined. Within the given model, $\log(G(A))$ has a smooth systematic behavior as a function of the model space dimension, as expected for an effective coupling strength. In particular, for the Pb and Sn isotopes $G(A)$ can be described by a two parameter expression that is inversely proportional to the dimension of the model space.

PACS. 21.10.Dr Binding energies – 71.10.Li Pairing interactions in model systems – 21.60.Cs Shell model

Exactly solvable models are important for gaining a deeper understanding of physical phenomena. For example, the Elliott SU(3) model bridges between the collective and microscopic shell-model descriptions of nuclei. Pairing is another exactly solvable model. Most such theories use Hamiltonians that are restricted to no more than two-body interactions. However, for many applications the infiniteness of the Hilbert space represents a serious obstacle, especially if the system is not exactly solvable or falls into a pertubative regime. To overcome this problem, one has to restrict the model space to a finite dimensional subspace and construct an appropriate effective Hamiltonian for that subspace. But this, in turn, leads to an interaction with many-body terms. Nonetheless, the effective Hamiltonian approach has been applied with increasing success, and even has pointed to the importance of three-body nuclear interactions [2]. Therefore, the study of Hamiltonians with many-body interactions can enhance our understanding of physical systems. The recently introduced exactly solvable extended pairing model [1] is such a many-body theory:

$$\hat{H} = \sum_{j=1}^{p} \epsilon_j n_j - G \sum_{i,j=1}^{p} B_i^+ B_j - G \sum_{\mu=2}^{p} \frac{1}{(\mu!)^2} \times \times \sum_{i_1 \neq \cdots \neq i_{2_\mu}} B_{i_1}^+ \cdots B_{i_\mu}^+ B_{i_{\mu+1}} \cdots B_{i_{2_\mu}}. \quad (1)$$

The standard Nilsson plus pairing Hamiltonian consists of the first two terms of (1). Besides these two, the Hamiltonian includes many-pair interactions which connect configurations that differ from one another by more than a single pair. Here p is the total number of single-particle levels considered, ϵ_j are single-particle energies, G is the overall pairing strength ($G > 0$), $n_j = c_{j\uparrow}^\dagger c_{j\uparrow} + c_{j\downarrow}^\dagger c_{j\downarrow}$ is the number operator for the j-th single-particle level, $B_i^+ = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$ ($B_i = (B_i^+)^\dagger = c_{i\downarrow} c_{i\uparrow}$) are pair creation (annihilation) operators where $c_{j\uparrow}^\dagger$ ($c_{j\downarrow}^\dagger$) creates a fermion in the j-th single-particle level. The up and down arrows refer to time-reversed states. Since each Nilsson level can only be occupied by one pair due to the Pauli Exclusion Principle, the operators B_i^+, B_i, and n_i form a hard-core boson algebra: $[B_i, B_j^+] = \delta_{ij}(1 - n_i)$, $[B_i^+, B_j^+] = 0 = (B_i^+)^2$.

The pairing vacuum state $|j_1, \cdots, j_m\rangle$ is defined so that: $B_i|j_1, \cdots, j_m\rangle = 0$ for $1 \leq i \leq p$ and $i \neq j_s$, where $j_1, \cdots, j_m$ indicate those m levels that are occupied by unpaired nucleons. Any state that is occupied by a single nucleon is blocked to the hard-core bosons due to the Pauli principle. The k-pair eigenstates of (1) have the form:

$$|k; \zeta; j_1 \cdots j_m\rangle = \sum_{i_1 < \cdots < i_k} C_{i_1 \cdots i_k}^{(\zeta)} B_{i_1}^+ \cdots B_{i_k}^+ |j_1 \cdots j_m\rangle, \quad (2)$$

where $C_{i_1 i_2 \cdots i_k}^{(\zeta)}$ are expansion coefficients that need to be determined, and the strict ordering to the indices $i_1, \cdots, i_k$ reminds us that double occupation is not allowed. It is always assumed that the level indices $j_1, \cdots, j_m$ should be excluded from the summation in (2). Since the general formalism is similar, we will focus only on the seniority zero case ($m = 0$).

Although Hamiltonian (1) contains many-body interaction terms that are non-perturbative, the contribution of the higher and higher energy configurations is more and more suppressed due to the structure of the equation that needs to be solved (5) to determine the eigensystem of the Hamiltonian (1). The eigensystem, $E_k^{(\zeta)}$ and $C_{i_1 i_2 \cdots i_k}^{(\zeta)}$,

depends on one and only one parameter $z^{(\zeta)}$:

$$E_k^{(\zeta)} = z^{(\zeta)} - G(k-1), \tag{3}$$

$$C^{(\zeta)}_{i_1 i_2 \cdots i_k} = \frac{1}{z^{(\zeta)} - E_{i_1 \ldots i_k}}, \quad E_{i_1 \ldots i_k} = \sum_{\mu=1}^{k} 2\epsilon_{i_\mu}, \tag{4}$$

$$1 = \sum_{i_1 < i_2 < \cdots < i_k} \frac{G}{E_{i_1 \ldots i_k} - z^{(\zeta)}}. \tag{5}$$

The additional quantum number ζ can be understood as the ζ-th solution of (5). Similar results for many broken-pair systems can be derived by using this approach except that the indexes j_s of the level occupied by the single nucleons should be excluded from the summation in (2) and the single-particle energy term ϵ_{j_s} from the first part of (1) should be added to the total eigenenergy. The eigenstates (2) are not normalized, but can be normalized once the coefficients $C^{(\zeta)}_{i_1 i_2 \cdots i_k}$ are known. The eigenstates (2) with different roots given by (5) are mutually orthogonal since they correspond to eigenstates with different eigenvalues.

For our Nilsson plus extended pairing model calculations, the single-particle energies were calculated using the Nilsson deformed shell model with deformation parameters taken from [3]. Experimental binding energies were taken from reference [4]. Theoretical binding energies were calculated relative to a specific core, ^{152}Yb, ^{100}Sn, and ^{208}Pb. The binding energy of the nucleus next to the core was used to determine an energy scale for the Nilsson single-particle energies. For an even number of neutrons, we considered only pairs of particles (hard bosons). For an odd number of neutrons, we apply Pauli blocking of the Fermi level of the last unpaired fermion and considered the remaining fermions as if they were an even A fermion system. The valence model space consists of the neutron single-particle levels between two closed shells with magic numbers 50-82 and 82-126. By using (3) and (5), values of G were calculated so that the experimental and theoretical binding energy match exactly. This means that the structure of the model space is encoded in $G(A)$.

Figure 1 shows results for the $^{181-202}$Pb isotopes. The binding energies are relative to ^{208}Pb, and the inert core nucleus was chosen to be ^{164}Pb. For the Yb and Sn isotopes the inert core nucleus was also the binding energy reference nucleus (^{100}Sn and ^{152}Yb). In this respect, the calculations for the Pb-isotopes are different because the inert core nucleus (^{164}Pb) and the binding energy reference nucleus (^{208}Pb) are not the same. One can see from Figure 1 that a quadratic fit to $\ln(G)$ as function of A fits the data well. In this particular case, the pairing strength $G(A)$ for all 21 nuclei in the range A=181 − 202) was also fit to a simple two-parameter function that is inversely proportional to the dimension of the model space $\dim(A)$, namely, by $G(A) = \alpha/[\dim(A)]^{\beta}$. Similar results have been obtained for the Sn-isotopes as well.

In conclusion, we studied binding energies of nuclei in three isotopic chains, $^{100-130}$Sn, $^{152-181}$Yb, and $^{181-202}$Pb, within the recently proposed extended pairing model [1] by using Nilsson single-particle energies as the input mean-field energies. Overall, the results suggest that the model is applicable to well-deformed nuclei if the pairing strength is allowed to change as a smooth function of the model space dimension. In particular, in all the cases studied $\ln(G(A))$ has a smooth quadratic behavior for even and odd A with a minimum in the middle of the shell where the model space dimension has a maximum; $\ln(G(A))$ for even A and odd A are very similar which suggests that further detailed analysis may result in the same $G(A)$ functional form for even A and odd A isotopes as found in the case of the Pb-isotopes and Sn-isotopes.

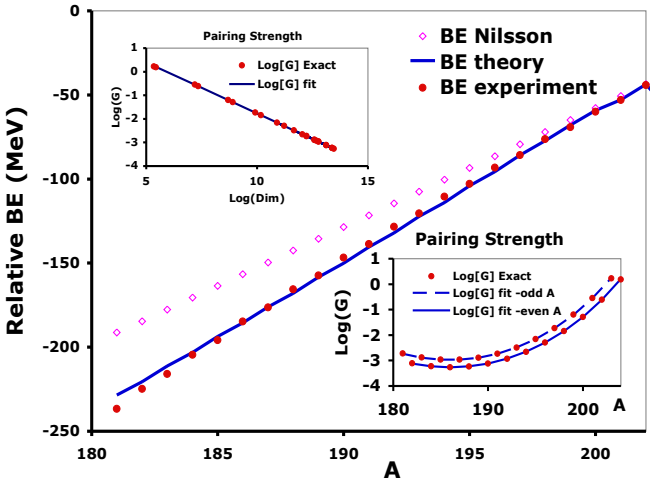

Fig. 1. The solid line gives the theoretical binding energy for the Pb isotopes relative to the ^{208}Pb nucleus. The insets show the fit to the values of G that reproduce exactly the experimental data using ^{164}Pb inert core. The lower inset shows the two fitting functions: $\log(G(A)) = 382.3502 - 4.1375A + 0.0111A^2$ for even values of A and $\log(G(A)) = 391.6113 - 4.2374A + 0.0114A^2$ for odd values of A. The upper inset shows a fit to $G(A)$ that is inversely proportional to the size of the model space, $(\dim(A))$, that is valid for even as well as odd values of A: $G(A) = 366.7702/\dim(A)^{0.9972}$. The Nilsson BE energy is the lowest energy of the non-interacting system.

Financial support provided by the U.S. National Science Foundation, the Natural Science Foundation of China, and the Education Department of Liaoning Province. Some of the work was also performed under the auspices of the U. S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

References

1. F. Pan, V.G. Gueorguiev, and J.P. Draayer, Phys. Rev. Lett. **92**, 112503 (2004).
2. P. Navratil and W. E. Ormand, Phys. Rev. C **68**, 034305 (2003).
3. P. Möller, J. R. Nix and K. L. Kratz, Atomic Data Nucl. Data Tables **66**, 131 (1997).
4. G. Audi, O. Bersillon, J. Blachot and A. H. Wapstra, Nucl. Phys. **A624**, 1 (1997).